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Rovelli's World

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Abstract Carlo Rovelli's inspiring "Relational Quantum Mechanics" serves several aims at once: it provides a new vision of what the world of quantum mechanics is like, and it offers a program to derive the theory's formalism from a set of simple postulates pertaining to information processing. I propose here to concentrate entirely on the former, to explore the world of quantum mechanics as Rovelli depicts it. It is a fascinating world in part because of Rovelli's reliance on the information-theory approach to the foundations of quantum mechanics, and in part because its presentation involves taking sides on a fundamental divide within philosophy itself.

Keywords Carlo Rovelli · Einstein–Podolski–Rosen · Quantum information · Relational quantum mechanics

Rovelli's inspiring "Relational Quantum Mechanics" provides an original vision of what the world of quantum mechanics is like.[1] It is fascinating in part because its presentation involves taking sides on a fundamental divide within philosophy itself.

[1] Rovelli [11]; I will refer by section numbers, since a revised version is available on the web.

I happily dedicate this paper to Jeffrey Bub, whose work has inspired me for a good quarter of a century.

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1 Placing Rovelli

1.1 Rovelli's Description of Rovelli's World

In Rovelli's world there are no observer-independent states, nor observer-independent values of physical quantities. A system has one state relative to a given observer, and a different state relative to another observer. An observable has one value relative to one observer, and a different value relative to another observer. (The relativity of values of observables follows from the relativity of states in this view, because Rovelli emphatically retains the 'eigenstate-eigenvalue link': observable A has value x precisely if the system to which A pertains is in an eigenstate of A. However, this must be read so as to accommodate 'vague' ascriptions of values, since the states of a system relative to various observers are generally mixed rather than pure.) 'Observer' does not have connotations of humanity or consciousness here—each system provides its own frame of reference relative to which states and values can be assigned. The analogy being drawn on continues a convention adopted at the birth of the theory of relativity, where observers were equated to moving spatial frames of reference.

We want to ask at once: what are the absolutes, the invariants, the features that do characterize these systems, in ways that are not relative to something else? That remains crucial to the understanding of this view of the quantum world. Following Rovelli's own convenient fiction of observers who measure and assign states to the objects they measure, we can think of those observers as having assimilated Rovelli's view, and thus having available some of his observer-independent description of what is going on. In assigning a state to a measured object, which includes information about probabilities of outcomes of possible future measurements, the observer draws on stable observer-independent features (notably, the algebra of observables and the 'transition probabilities' provided by quantum mechanics).

1.2 History of Quantum Theory Interpretation

We can relate Rovelli's approach to a fundamental division among interpretations of quantum mechanics that was outlined by John Wheeler. When Everett published his seminal paper in 1957, Wheeler added a commentary acknowledging that throughout the history of Quantum Mechanics so far, there had been two views in tension with each other, and he argued that Everett had finally made the 'one true story of the universe' version feasible:

> (1) The conceptual scheme of "relative state" quantum mechanics is completely different from the conceptual scheme of the conventional "external observation" form of quantum mechanics and (2) The conclusions from the new treatment correspond completely in familiar cases to the conclusions from the usual analysis. The rest of this note seeks to stress this *correspondence in conclusions* but also this *complete difference in concept.* [16, p. 463]

Wheeler is here contrasting Everett's conception with the older 'external observation' conception, that he describes as follows:

> The "external observation" formulation of quantum mechanics has the great merit that it is dualistic. It associates a state function with the system under study—as for example a particle—but not with the *ultimate* observing equipment. The system under study can be enlarged to include the original object as a subsystem and also a piece of observing equipment—such as a Geiger counter—as another subsystem. At the same time the number of variables in the state function has to be enlarged accordingly. However, the *ultimate* observing equipment still lies outside the system that is treated by a wave equation. (1957, ibid.)

Rovelli clearly places himself in the older 'external observation' formulation, opposite to the new one that Wheeler lauds. But there is one very important difference that places Rovelli somewhat nearer Everett's. Rovelli takes seriously the idea that any and every system can play the role of 'ultimate observing equipment':

> By using the word "observer" I do not make any reference to conscious, animate, or computing, or in any other manner special, system. I use the word "observer" in the sense in which it is conventionally used in Galilean relativity when we say that an object has a velocity "with respect to a certain observer". The observer can be any physical object having a definite state of motion. For instance, I say that my hand moves at a velocity v with respect to the lamp on my table. Velocity is a relational notion (in Galilean as well as in special relativistic physics), and thus it is always (explicitly or implicitly) referred to something; it is traditional to denote this something as the observer, but it is important in the following discussion to keep in mind that the observer can be a table lamp. (end sect. I.)

Thus Rovelli insists that all systems "are assumed to be equivalent, there is no observer–observed distinction". In saying this he does not take back his rejection of the notion of observer-independent states or observer-independent values of physical quantities. Instead, he means that just as in his guiding example of relativity theory, every physical object can be taken as defining a frame of reference to which all values of physical quantities are referred. Related to this objectification of the 'external observer' is his conception of *information* in physics:

> Also, I use information theory in its information-theory meaning (Shannon): information is a measure of the number of states in which a system can be—or in which several systems whose states are physically constrained (correlated) can be. Thus, a pen on my table has information because it points in this or that direction. We do not need a human being, a cat, or a computer, to make use of this notion of information. (ibid.)

Rovelli takes it that any system can in principle have information about any other, due to a previous interaction, for he equates the having of information in its physical sense with a correlation that has been effected by such an interaction:

> any physical system may contain information about another physical system. For instance if we have two spin-$1/2$ particles that have the same value of the spin in the same direction, we say that one has information about the other one.

> Thus observer system in this paper is any possible physical system (with more than one state). If there is any hope of understanding how a system may behave as observer without renouncing the postulate that all systems are equivalent, then the same kind of processes—"collapse"—that happens between an electron and a CERN machine, may also happen between an electron and another electron. Observers are not "physically special systems" in any sense.

We must treat this with some delicacy, since the usual explanation of such correlations or entanglements is in terms of states conceived of as observer-independent. The standard quantum mechanical formalism is used here, but understood in a new way.

Given the comparative loss of popularity of the older 'external observation' approach, at least among those who work on foundations of physics, Rovelli's return to it at this date imparts his view with a stimulating sense of novelty.

1.3 Information-Theory Approach from Groenewold to the Present

Noting the emphasis Rovelli puts on information, it is also important to place Rovelli's approach with respect to the information-theory approach. This is a very lively new development. While there were beginnings and precedents, this has recently taken a quite radical turn, and Rovelli's work can be seen as involved in that turn. Let's look at the beginnings first and then at the radical agenda in such recent work as that of Christopher Fuchs, Jeffrey Bub and their collaborators.

In the 1950s H.J. Groenewold advocated that we should regard quantum states as just summaries of information obtained through measurement. There are some striking similarities between Groenewold's description of the quantum mechanical situation and Rovelli's.

Groenewold [6, 7] proposed a formulation of the theory that would contain all its empirical content without referring to states in any essential way. He derided the idea that quantum states are to be thought of on the model of states in classical mechanics. His formulation re-appears quite clearly in Rovelli's article, though there in a more general form. The idea is that a situation of interest is to be depicted as the effect of a series of measurements, represented by a series of observables (the ones being measured) interspersed with evolution operators (governing evolution between measurements). The sole real problem to be addressed, according to Groenewold, is this:

> given the outcomes of preceding measurements, what are the probabilities for outcomes of later measurements in the series?

The answer is formulated in terms of transition probabilities.[2] In the exposition of Rovelli's specific version below I shall explain and illustrate how that goes.

Groenewold offers an argument to the effect that states are to be regarded as 'subjective' or 'observer-relative', determined by information available. Imagine that each measurement apparatus in the series records its outcome.[3] After the entire series has

[2]Groenewold was not the only one; see for example [13].

[3]See Dicke [3] for an argument about how this is physically possible without disturbance; see further the discussion in [15, pp. 257–258].

been concluded, a physicist O inspects those recorded results in some order, and assigns states to the system measured for the times of those outcomes using von Neumann's Projection Postulate recipe (which everyone agrees is fine for such narrowly focused predictive tasks). To begin, O assumes some initial state. Groenewold suggests that in absence of other information that could be the entirely uninformative mixture represented by the identity operator on the space. For time t between times t_1 and t_2 where the state $\rho(t_1)$ is assumed or known the calculation looks like this:

$$\rho(t) = U(t, t_2)K(t_2)\rho(t_1)K(t_2)U(t_2, t)$$

(with a correspondingly longer such series for a longer series of measurements between the initial and final time) where the Ks are transition operators, and the time-indexed ρ is the ascribed state; the Us are the normal evolutions while no measurement or other interference occurs.

But now what would happen if O (or one of his colleagues) decides on a different order for inspection of the recorded outcomes? For the same times, although having started with the same initial knowledge or assumptions about the system, the assignment of states will be quite different.

There is nothing contentious in this imagined scenario itself. The contentious part is Groenewold's insistence that no other significance is to be accorded to the assignment of states. They are nothing more than compendia of information assumed, known, or gathered through measurements, and thus determined entirely by a specific history, the 'observer's' history. The truly empirically testable part of the theory, he insists, is contained in the transition probabilities. When they are tested, the convenient calculation starts with an assignment of an initial state, but coherence requires only that some such initial assignment leads to the right predictions—the transition probabilities are independent of the states, they are formulable in terms of the observables.[4]

This insistence, that the states be thought of as playing no other role, is at the heart of the recent innovations in the information theoretic approach. Christopher Fuchs presents the program in its most radical form in his much discussed "Quantum Mechanics as Quantum Information (and only a little more)":

> This, I see as the line of attack we should pursue with relentless consistency: The quantum system represents something real and independent of us; the quantum state represents a collection of subjective degrees of belief about something to do with that system (even if only in connection with our experimental kicks to it). The structure called quantum mechanics is about the interplay of these two things—the subjective and the objective [5, p. 5].)

He submits that "the quantum state is solely an expression of subjective information—the information one has about a quantum system. It has no objective reality in and

[4]They are often presented as probabilities for transitions between states, because the Projection Postulate is generally taken for granted. In [15] I explained them in an intermediate way: the probability is that of the outcome 1 of a measurement of the observable represented by projection on the vector representing second state, given that the system measured is in the first state. But this can easily be replaced by a formulation in terms of the two observables, which are the projections on the two states. For a nice introductory treatment of the theory entirely in this form we can look to [13].

of itself." When asked "information about what?" he replies "The answer is 'the potential consequences of our experimental interventions into nature'." (ibid, p. 7) But Fuchs also has a precise proposal about how to describe the information-updating process in response to measurement. (See further [4].) Drawing on results, both his own and others, he depicts it as a special case of Bayesian updating of opinion by conditionalization. We have to think here, as in Groenewold's scenario, of an epistemic agent with a pertinent prior state of opinion—a physicist who accepts at least the bare minimum of the quantum theory—reacting to recorded measurement outcomes. There is also, without explicit attention paid, for both Groenewold and Fuchs, a presumed coordination, so that tangible physical operations can be univocally represented in terms of an algebra of observables of a certain sort.

This reliance on a fundamental representation of the physical situation—the coordination—becomes clearest in the important paper by Robert Clifton, Jeffrey Bub, and Hans Halvorson. The physical system is characterized by means of an algebra of observables, taken to be a C* algebra.[5] But states are just generalized probability functions—more accurately, expectation value functions—defined on this algebra of observables. So far that is similar to the approach in more "realistically" understood foundational treatments. The difference comes in what is added now so as to single out quantum theories. What is added is constraints on information transfer, with the states thought of as information depositories. From the premise that those constraints are satisfied, the basic principles of quantum theory are deduced. As reflection on this result, Bub then argued in his "Why the Quantum?" that

> A quantum theory is best understood as a theory about the possibilities and impossibilities of information transfer, as opposed to a theory about the mechanics of non-classical waves or particles. [1, p. 42]

"Information" is here understood as Groenewold specified, in the technical sense of information theory, as measured classically by the Shannon entropy or by the von Neumann entropy for quantum states. And in "Quantum Mechanics Is About Quantum Information", Bub argues that

> Quantum mechanics represents the discovery that there are new sorts of information sources and communication channels in nature (represented by quantum states), and the theory is about the properties of these information sources and communication channels. You can, if you like, tell a mechanical story about quantum phenomena ... but such a story, if constrained by the information-theoretic principles, will have no excess empirical content over quantum mechanics. So the mechanical story for quantum phenomena is like an aether story for electromagnetic fields. [2, p. 558]

Bub's answer to the question "Information about what?" is just the same as Fuch's—though in phrasing that shows his special interest in encryption and decoding.

Note once again that some form of coordination is presumed given, without receiving explicit attention: the measurements and their results are assumed univocally

[5]This is a very general framework, which allows for the formulation of many sorts of physical theories, both classical and quantum.

representable in terms of the observables that characterize the system. This points to 'absolute' characteristics of the system, which are not aspects of information gathered about it, but pertain to the system itself. That the system is characterizable in such a way is presupposed when certain operations are classified as, or taken to be, means of gathering information about it. Thus here, as for Groenewold (and equally for Rovelli, as we shall see) there is a divide as well as a link between 'subjective' and 'objective' features of the experimental situation.

2 Is There a View from Nowhere?

At first sight Rovelli's treatment of states is not exactly what either Groenewold, Fuchs, or Bub appears to advocate.[6] Rovelli does bring states into the discussion, but as states that measured objects can have *relative to* the measuring system. At first sight we seem to detect a tension between what Rovelli does and what he tells us it is possible to do. What he calls his Main Observation, motivating the view, is similar to Groenewold's though:

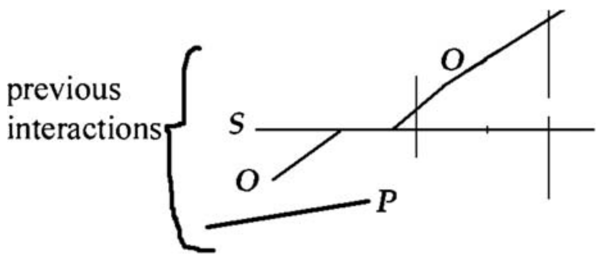
> In quantum mechanics different observers may give different accounts of the same sequence of events.

Having rejected the idea of observer-independent states, there is no question of one of those descriptions being the sole truth, with the other illusion or error. Here is an example that Rovelli describes in intuitive terms. I will elaborate on it, in several steps.

Example 1 The two-observer situation

To begin we can characterize it as follows.

There are two observers, O and P, and one other system, S. Observer O measures an observable A on system S, while the second observer P describes this measurement by O on S. (Later on P may make a measurement on S too, or on $S + O$; but we will leave this unexplored for now.)



O registers the value 1, say, and thus assigns pure state $|A, 1\rangle$ to S, or in other words, S is now in state $|A, 1\rangle$ relative to O.

[6] Though Rovelli's article was clearly inspirational to the later literature; cf. Fuchs, op. cit. p. 3.

Meanwhile P has the information that this measurement is taking place (presumably on the basis of earlier measurements made on $S + O$). So P describes O as in an initial state $|init\rangle$ and $S + O$ coupled at the beginning. The state of $S + O$ evolves:

$$(\alpha|A, 1\rangle + \beta|A, 0\rangle) \otimes |init\rangle \rightarrow (\alpha|A, 1\rangle \otimes |B, 1\rangle) + (\beta|A, 0\rangle \otimes |B, 0\rangle).$$

Here $|B, 1\rangle$ and $|B, 0\rangle$ are the 'pointer reading states' that P uses to characterize observer O when O registers a definite value of 1 or 0 as measurement outcome. That is, the measurement interaction between S and O is such as to effect the requisite correlation between A pertaining to S and B pertaining to O.

If P now wonders what state to assign to S, but does not make a measurement, then he calculates it by the usual 'reduction of the density matrix'. Thus P assigns to S a mixed state, namely the mixture of $|A, 1\rangle$ and $|A, 0\rangle$ in proportions α^2 and β^2. So we see that O and P assign different states to S. To put it in other words, S has different states relative to O and relative to P.

Rovelli also insists on the orthodox eigenvalue-eigenstate link, so that A takes a value 1 relative to O, but not relative to P—observables have values only relative to observers, and may not have the same value relative to different observers.

But is *this* description of the situation then observer-independent, one that is in fact not relative to any observer? Shouldn't we object that the rationale forbids this, because by Rovelli's lights we can only have descriptions relative to some observer or other?

2.1 General form Versus Third-Observer Description

The answer is that there is no incoherence here, but we must carefully distinguish what Rovelli gives us when he presents his view, even in such an example, and the description of the same situation by a third observer. The Example can indeed be elaborated so as to include a third observer, whom we might call ROV. We could imagine that ROV has, on the basis of previous measurements, information that can be summarized by assignments of initial states to O, S, P and their composites relative to ROV, plus later states based on their unitary evolution. We'll look later at how this goes, when we will also have occasion to consider measurements that P can make on O or S later on. But right now we can point out that ROV's information is not to be confused with what Rovelli tells us about this sort of situation. The tension that a reader might feel could be expressed this way:

> Rovelli seemingly purports to be giving us a description of the world that would on the one hand be *on the same level as* a description of the rest of the world relative to some given system ROV, and yet on the other hand not relative to anything!

But that is not so at all. Rovelli, who can give these examples, is telling us only something about the *general form* that these observers' descriptions (their information) can take, given that certain measurement interactions have taken place. The resolution of this sensed tension is this: Rovelli does not give any specific such description of the world—he describes *the form that any description which assigns states must take.*

Rovelli describes not the world, but the general form of information that one system can have about another—namely as the assignment of states relative to a given system on the basis of information available to that system:

- there is no implication of possible specific information about what there is which is independent of any point of view, but
- there can be knowledge of the form that any such information, relative to a particular vantage point, must take.

So we have here a *transcendental* point of view. Rovelli offers us this knowledge of the general form, the conditions of possibility. We must take very seriously the fact that as he sees it, quantum mechanics is not a theory about physical states, but about ('about'?) information. The principles he sees at the basis of quantum mechanics are principles constraining the general form that such information can take, not to be assimilated to classical evolution-of-physical-state laws.

2.2 The form of an Observer's Description of the World

This form is constrained by the insistence that specific information, had by one system about another system, can only be a record of actual measurement outcomes. The only way in which there can be information for one observer of what has happened to another observer is through a physical measurement by the former on the latter. Communication, i.e. exchange of information, is physical (cf. end sect. III of the article).

Before aiming at greater precision, let's briefly summarize how this happens according to Rovelli's account. A *question* is asked of a system or source only when an appropriate physical interaction takes place. This interaction is a *measurement* delivering a value for some observable, but also serves as a *preparation*, so that the value obtained has (relative to the theory) predictive content. The probabilities of future measurement outcomes are affected by the outcome obtained—the measured system has gone into a new state relative to the measurement set-up. Thus he accepts (explicitly, in his rejection of the Bohm and modal interpretations) von Neumann's *eigenstate-eigenvalue link*:

> the system to which the observable's value pertains is (at that time) in an eigenstate of that observable, corresponding to that value.

But there is a twist, which changes the meaning, so that this says something quite different from its original. The reference is here not to a physical state of the system, but to the state of the system relative to the observer (the measurement apparatus). So the 'collapse' is in that observer's information; the state assigned to the system is a summary of that information.

As mentioned earlier, because of the eigenstate-eigenvalue link it follows that if states are relative, so are values of observables. That an observable takes or has a certain value at a certain moment, that too is observable-relative (cf. end of sect. 2 [10]). Because information can only be had by actual, physical measurement, the states assigned will rarely be pure. It is not easy to obtain maximal information about a system, even with respect to targeted observables. So in general the value of an observable, relative to a given observer, will not be sharp.

This information is the subject of two postulates. Let us introduce them in such a way as to spell out what is and is not observer relative. Each physical system S is characterized in the first place by means of a set $W(S) = \{Q_i : i \text{ in } I\}$ of questions that can be asked of it. This association of $W(S)$ with S is not relative to any observer—we may call it the first 'absolute'. Although the presentation differs, this set of questions pertaining to S is essentially the specification of the family of observables that pertain to S. (Eventually, the algebra of observables is reconstructed from this family of questions; for our purposes we need not distinguish the two.) When the sets of questions are the same for two systems we may call them *of the same type*.

Secondly, an observer who has been in measurement interaction with a system has a record of the questions that have been asked and the sequence of outcomes thus obtained. That the observer has this *is not relative* to another observer.[7] It is our second 'absolute'. At the same time we must be careful not to equate this fact about the observer with a quantum mechanical state! For while we could try to describe a state that ostensibly is the state that O has if and only if it has a particular sequence of 0s and 1s registered in a series of measurement interactions with S, that would have to be the state of O relative to another observer P who has obtained that information by means of a later measurement on S. We'll see later on whether, or to what extent, there could be a discrepancy, or even a meaningful comparison.

3 States as Observer-Information

3.1 The Postulates Constraining Information Acquisition

> Postulate 1 (Limited information) There is a maximum amount of relevant information that can be extracted from a system.

Answers to questions have predictive value, but typically, earlier answers become ***irrelevant*** to the predictions after later answers, *and must do so*. "Irrelevant" and "redundant" are perhaps not entirely apt terms: if a state is to be assigned on the basis of the extracted information, earlier answers must typically have to be *discarded* from the basis on which states are assigned.

> For system S there is a definite probability that given question Q will get a yes-answer if asked; this probability can differ for another system of type $W(S)$; moreover, this probability is affected by the answers to previous questions asked.

The 'moreover' establishes that the probabilities in question are transition probabilities. This I will spell out further after the second postulate. Note that what these transition probabilities are is the same regardless of which observer O asks the questions of S. So we have here a third 'absolute'. But fourthly, if we look at how the probability of future measurement outcomes changes in the course of asking

[7] By taking this not to be relative, we have in this sequence of 0s and 1s something analogous to Einstein's local coincidences, the 'bed rock' of the representation. Rovelli's criticism of the 'consistent histories' interpretation suggest strongly that he does not allow any ambiguity in this respect.

$c = \langle Q_1, Q_2, Q_3, \ldots, \rangle$ and getting number sequence $s_c = [n_1, n_2, n_3, \ldots]_c$ the items that become irrelevant after a certain point *are also the same for all systems of the same type* (the fourth 'absolute'). So, given these notions, we can define:

> *maximally non-redundant question-answer sequence*: one in which no element is irrelevant, but which loses that feature if any question + answer at all is added.[8]

Postulate 1 says that this sequence is finite. In a particular case, we can ask for the relevant finite number: how many questions are needed to extract maximal information, leading to the assignment of a pure state relative to the observer? This number *does not depend on which sequence* of questions we pick, and hence also is not relative. Thus Rovelli writes, in a passage immediately following Postulate 1:

> One may say that any system S has a maximal "information capacity" N, where N, an amount of information, is expressed in bits. This means that N bits of information exhaust everything we can say about S. Thus, each system is characterized by a number N. In terms of traditional notions, we can view N as the smallest integer such that $N \geq \log 2k$, where k is the dimension of the Hilbert space of the system S. Recall that the outcomes of the measurement of a complete set of commuting observables, characterizes the state, and in a system described by a $k = 2N$ dimensional Hilbert space such measurements distinguish one outcome out of $2N$ alternatives (the number of orthogonal basis vectors): this means that one gains information N on the system.

The number in question therefore depends on the dimension of the state space—if that dimension is finite number k then N is $\log_2 k$ or just above (to make N an integer); the dimension is 2^N, or $(2^N) - 1$. We have a good link here with information theory: the missing information, about what this 'source' of type $W(S)$ is like, is extractable in at most N Yes–No questions: the *maximal information capacity* of a system (source) of this type is N bits. [9] But now Rovelli adds:

> Postulate 2 (Unlimited information) It is always possible to acquire new information about a system.

This is not at odds with the first postulate, given that new information can make older information 'irrelevant' (having to be discarded). But it is certainly at odds with the classical ideal of perfectible measurement, as revelation of aspects of the state of the system before measurement, without affecting that state. It entails a certain degree of indeterminism: the maximum possible information at a point does not settle what new information we could get. That is in part because observables can be (totally) *incompatible*: they may have no joint eigen-state:

> given a Yes answer to question Q there are many questions Q' such that if they are then asked, their answer *cannot* be Yes with certainty, nor No with certainty.

[8]Rovelli introduces and uses the term "complete family s_c of information" for "*maximally non-redundant question-answer sequence*".

[9]Compare: "In particular, I identify one element of quantum mechanics that I would not label a subjective term in the theory; it is the integer parameter D traditionally ascribed to a quantum system via its Hilbert-space dimension." Chris Fuchs [5], Abstract.

Notice the modal character of this assertion! In contrast, some questions Q and Q' are *compatible*: on a given occasion, after receiving Yes to Q, the observer has only non-zero probabilities for both possible answers to Q', but if he then asks Q', he can base more precise predictions on the fact that he has had these two answers.

> *New assumption*: this indeterminism is not a chaotic randomness, but can be characterized in terms of definite probabilities.

Suppose the first complex apparatus A asks a "complete" question, so it yields a record that provides a *maximally non-redundant question-answer sequence*. Before that question has been asked we have no non-trivial information. Suppose the second apparatus B is equally *complete*, though the question family is very different. Rovelli posits a definite *transition probability* $p(B|A)$ that a Yes answer to B will follow a Yes answer to A, which is both idempotent and symmetric.

Intuitive mnemonics: look at the scenario in which a single source sends many systems of the same type into the series of measurement apparatus for two-valued observables $A, B, \ldots$ that the observer has installed. The stream is diminished by some factor q by the first measurement, then by the transition probability p. Suppose we do A again, then once again the stream is diminished by that factor p. So the number goes from qM to pqM to $ppqM$ by the operations A, AB, ABA so we could write:

$$ABA = pA$$

and this is what a sequence of 1-dimensional projections would do to a vector. It is a way to identify the transition probability. This is numerically equal to the $\cos^2$ of the angle between the two 1-eigenvectors, onto which they project, or in Hilbert space the squared modulus of the scalar product, or equivalently the trace of the product of the two projections.

After a *maximally non-redundant question-answer sequence* performed by measurement A, the next question might only e.g. ask "is the system in subspace J?", with J of higher dimension—but here there is a definite probability as well, which can be derived (in accordance with the practical calculation suggested by von Neumann's Projection Postulate).

3.2 States as States of Information, Relative to the Observer

Suppose that observer O has put a series of questions to system S and has arrived at the point of attributing $|A, x\rangle$ to S, where x is an eigen-value of A. Imagine once again a second observer P, whose knowledge (gained earlier through a physical transmission process) was enough to attribute an initial state to $S + O$, and a Hamiltonian to govern their interaction, enough for him to attribute the evolution in question. Then as we noted above P has the usual 'distant' description of $S + O$:

$$\text{initially it is in state} \qquad \sum_i \beta_i (|A, a_i\rangle \otimes |init\rangle) \tag{1}$$

$$\text{this evolves into the final state} \qquad \sum_i \beta_i (|A, a_i\rangle \otimes |B, a_i\rangle) \tag{2}$$

where B is the 'pointer observable' of O—its value being a recorded sequence of 0s and 1s. Using a reduction, P can attribute a state to S as well, namely

$$\text{a mixture of states } |A, a_i\rangle \text{ with weights } \beta_i^2$$

which is quite different from $|A,1\rangle$ or $|A,0\rangle$. According to Rovelli, this is all there is to be said, so far: S has one state relative to O, and another state relative to P. The phrase 'S has state $|A,1\rangle$ relative to O' means only that the information O has obtained can be summed up or represented by the vector $|A,1\rangle$. But is the fact that O has certain information a fact that is or is not observer-relative? We must answer this question in the light of two points Rovelli insists on:

(i) There is no meaning to the state of a system except within the information of a further observer.
(ii) There is no way a system P may get information about a system O without physically interacting with it, and therefore without breaking down (at the time of the interaction) the unitary evolution description of O.

'Information' has a minimal sense in this context, to say that O has information about S means only that there is a certain correlation in the state of $S + O$. That much P was able to predict already, and so he can predict something with certainty if a measurement will be made to confirm this. Note that what he is able to predict with certainty amounts to information he already has.

More formally, there is an operator M on the Hilbert space of the $S + O$ system whose physical interpretation is "Is the pointer correctly correlated to A?" If P measures M, then the outcome of this measurement would be yes with certainty, when the state of the $S + O$ system is as in the state described in (2). The operator M is given by

$$\begin{aligned} M(|A,1\rangle \otimes |B,1\rangle) &= |A,1\rangle \otimes |B,1\rangle \\ M(|A,1\rangle \otimes |B,0\rangle) &= 0 \\ M(|A,0\rangle \otimes |B,0\rangle) &= |A,0\rangle \otimes |B,0\rangle \\ M(|A,0\rangle \otimes |B,1\rangle) &= 0 \end{aligned} \tag{3}$$

where the eigenvalue 1 of M means "yes, the hand of O indicates the correct state of S" and the eigenvalue 0 means "no, the hand of O does not indicate the correct state of S". At time $t2$, the $S + O$ system is in an eigenstate of M with eigenvalue 1; therefore P can predict with certainty that O "knows" the value of A.

Thus, it is meaningful to say, according to the P description of the events E, that O "knows" the quantity A of S, or that O "has measured" the quantity A of S, and the pointer variable embodies the information (cf. middle of section II-D). But of course P had a choice, P could have measured a different observable, say K, to try and find out which result O obtained:

$$\begin{aligned} K(|A,1\rangle \otimes |B,1\rangle) &= |A,1\rangle \otimes |B,1\rangle \\ K(|A,1\rangle \otimes |B,0\rangle) &= 0 \end{aligned}$$

$$K(|A,0\rangle \otimes |B,0\rangle) = 0$$

$$K(|A,0\rangle \otimes |B,1\rangle) = |A,0\rangle \otimes |B,1\rangle$$

Intuitively speaking, this is what P would measure to find out what O found. She would get either result 1 or result 0, and would say "O found 1" or "O found 0" accordingly. But can we understand that literally as referring to what O had as information before P made this measurement? If P finds result 1, does that imply that O had found 1 and that O had assigned state $|A, 1\rangle$ to S?

According to Rovelli's rules, *this makes no sense*. An interpretation of quantum measurement as revealing pre-existing values is untenable.

We are now in a position to examine and resolve some puzzles that tend to occur to practically any reader in first acquaintance with this interpretation.

4 Puzzles Posed and Resolved

All the puzzles will pertain to this basic situation:

O has made a complete measurement on S of two-valued observable A, and has a record of the question asked (call it $?A$) and the answer received; say 1. Accordingly S has now state $|A, 1\rangle$ relative to O. The pointer observable on O is B, so on the old, pre-Rovelli view one takes it that the existence of the record means that B has value 1. For Rovelli this makes no sense as an observer-independent assertion. To mention values of the pointer observable at all, we need to look at O from the point of view of second observer P.

Meanwhile P had made earlier measurements on $O + S$ and so has the information throughout that this measurement interaction is taking or has taken place. Based on his earlier results and his predictions on that basis, $O + S$ has at the end of the interaction an entangled state, namely $(\beta_0|B,0\rangle \otimes |A,0\rangle) + (\beta_1|B,1\rangle \otimes |A,1\rangle)$, relative to P.

PUZZLE 1. Could O and P Contradict Each Other?

Suppose that P will make a measurement on $O + S$ after this point, and later report the result to O. In the meanwhile O makes a prediction with certainty about what P will find. Is it possible that O will find his prediction contradicted by P?

> Example: P will measure $(I \otimes A)$ on $O + S$. P predicts that he will get value 1 with probability < 1, and value 0 with some probability > 0. Suppose he gets value 0.
>
> Meanwhile O knows that he has seen value 1, and has a record of that, so assigns himself state $|B, 1\rangle$, and assigns to S the state $|A, 1\rangle$, and therefore to $O + S$ the state $|B, 1\rangle \otimes |A, 1\rangle$. So O predicts with certainty that P's measurement will have result 1. And so O is making a false prediction here, one that is falsified by what P finds.

REPLY: The reasoning is questionable in several ways.

To begin we may note an unwarranted assumption in the second paragraph: that O has a state relative to itself here. There was no self-measurement in the story. The relative states are only assigned as summaries of what the real measurement results have been. So as far as this story goes, O has here no state relative to itself, nor does $O + S$ have a state relative to O.

Nevertheless, we can leave aside the issue of whether the possibility of self-measurement could be added to Relational Quantum Mechanics, for there is a much more important point to be made.[10]

The more important point is this. It is not to be assumed that P will ever find 0 in the case in which O has found 1. The insinuation in the above puzzle is that, if this were so, then P's probabilities would be wrong—and since these probabilities come from quantum mechanics, that such a scenario would contradict quantum mechanics. But this threat disappears as soon as we take heed of what P's probabilities are. They are what he calculates on a basis that includes no information about what O found. These probabilities would be tested by placing P very often in a situation that matches the information he has. P's probabilities are for his finding value 1 or value 0 in a situation *of that sort*—where this sort is not identified in terms of what O finds during the process, but only in terms derivable from preceding measurements of O and S that established that an A measurement would take place. We can be sure that if quantum mechanics is right, and P enters into many such situations, he will find values 1 and 0 with the correct frequencies.

What we may note in addition (and to this we will return) is that any immediate attempt to check by measurement whether O's and P's outcomes were the same, would get a positive result.

PUZZLE 2. But What About 'Immediate Repetition' of Measurement?

As von Neumann emphasized, O will predict with certainty that a measurement of A on S, immediately after his own, will find the same value. So does that not apply here, to an immediately subsequent measurement by P?

> REPLY: No; in Rovelli's account the collapse of the wave packet appears only in the states relative to a given observer. So his echo of von Neumann is that O will predict with certainty that if *he himself*, or an observer with exactly the same interaction history with a system of type $W(S)$, makes an immediate new measurement of A on S, the same value 1 will appear again.

As Rovelli emphasizes, O can get to know P's result only through a relevant interaction with P, in effect a measurement by O on P. So O could ask the question: "what did P see, when he measured A on S after me?"—in the sense that O can measure P's pointer observable afterward, and get some value. As usual, we cannot assume that the result that O gets is the value that this observable had before O's measurement.

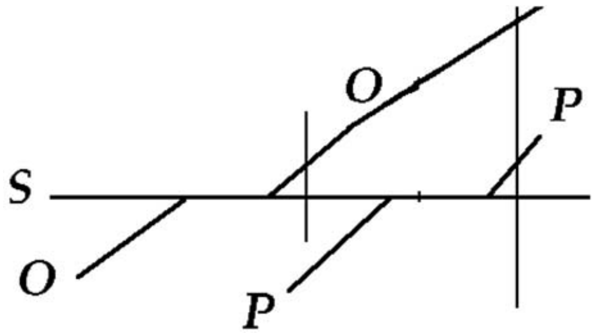
[10] My suggestion is that this should not be added as a possibility; there certainly seems to me to be no warrant in Rovelli's interpretation for doing so. For a contrary view and a recent 'Wigner's friend' type example presented to challenge information-theoretic approaches (specifically Jeffrey Bub's recent work) see [8].

So does this mean that O and P have no way to find out what either of them saw earlier, as opposed to what it seems now that they have seen? That would still seem almost as puzzling. To answer this properly, we need to construct our puzzle situation with more precision and care.

PUZZLE 3. Can an Observer Find out What was Observed Earlier on?

To see how we can get into a confusion here, I am going to present this third version of the puzzle first of all in the 'old' style, assuming that states are observer-independent. Then the puzzle will again be resolved by seeing how the understanding of this situation changes on Rovelli's conception.

Let the measured system S start off in a superposition $\sum \beta_{\mathrm{i}}|A, i\rangle$ of eigenstates of an observable A corresponding to distinct eigenvalues, and let us measure A twice, using two measuring systems O and P.[11] For simplicity I'll take A to be time-independent (we could put in evolution operators, as Groenewold and Rovelli indicate, but it would not seriously affect the argument), and take the pointer observables of both O and P to be the same observable B.



Then, under the familiar idealized assumptions of a von Neumann measurement, the combined system $S + O$ will be in dynamic state $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle)$ at the end of the first measurement. At the end of the second measurement the dynamic state of $S + O + P$ will be, ignoring phase factors, $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle \otimes |B, i\rangle)$.

By reduction, we have states also for parts of the total system. Write $P[|A, i\rangle]$ for the projection on the ray containing $|A, i\rangle$, etc. At the end of the first measurement, the individual systems S and O are in dynamic states $[\sum |\beta_i|^2 P[|A, i\rangle]$ and $\sum |\beta_i|^2 P[|B, i\rangle]$, respectively. The final dynamic state of $O + P$ is $\sum |\beta_i|^2 P[|B, i\rangle \otimes |B, i\rangle]$. Following von Neumann, ***assuming collapse***, we reason as follows:

> As for the individual states, because S and O interact by a measurement interaction, S ends up in some $|A, k\rangle$, with O in the corresponding $|B, k\rangle$. At the conclusion of the first measurement, the pointer reading observable B on O

[11] Assume that I and O each evolve freely after their measurement interaction, that there is no interaction between O and P, and that both A and the 'pointer-reading observable' B for O commute with the free Hamiltonians for S and O respectively.

thus has the value k as well, we will say that its pointer reads k. Likewise, at the end of the second measurement, S ends up in some state $|A, m\rangle$ with P in the corresponding $|B, m\rangle$; its pointer the reads m. Moreover, $m = k$.

Suppose we want to check now whether that is so. Then we can have a *third* measurement, of that 'agreement observable' that Rovelli describes, as follows:

Let M be an observable for the combined system $O + P$, which has eigenvalue 1 on the space spanned by all $|B, i\rangle \times |B, i\rangle$, and which has value 0 on all $|B, i\rangle \times |B, j\rangle$, for j not equal to i. Then if O and P are in pure dynamic states $|B, i\rangle$ and $|B, j\rangle$ respectively (always ignoring phase factors), the value of M will be 1 if and only if $i = j$. In the usual interpretation, this means that in the only case in which our pointer readings can have definite values, M will have the value 1 just in case these values agree. In the context of that interpretation, then, it is reasonable to speak of M as the observable which is, or registers, agreement between the two pointer readings. Even in the context of Rovelli's interpretation, one can continue to speak of M as the 'agreement' observable. The question is whether here, the locution needs to be taken with a grain of salt—may M take up the value 1 even though the pointer readings do not agree?

Our present example provides an illustration. The final dynamic state of $O + P$ is $\sum |\beta_i|^2 P[|B, i\rangle \times |B, i\rangle]$. Since all summands of the mixture are eigenstates of M with eigenvalue 1, so is the state itself. So M takes the value 1 on the system $O + P$. To arrive at this conclusion however, we needed only to know the mixed state here ascribed to $O + P$—we did not need any information about what states O, P are in individually. That information is logically compatible with the equally valid conclusion that O, P are in mixtures of the various states $\{|B, i\rangle\}$. So the conclusion that M takes value 1 cannot possibly, by itself, guarantee the suggestion that the pointer reading on P is equal to the pointer reading on O. But if we assume von Neumann's rather than Rovelli's interpretation, we do have that guarantee, since O, P collapsed into definite pointer states.

REPLY: Once again, we have drawn a puzzling consequence for Rovelli by thinking about the situation in 'old' terms, and then having too quick a look at how his view differs. To really see whether there is a puzzle here, we have to retell the story from the beginning, in Rovelli's way. Here is the retelling, which we can now exhibit as a more elaborate example of Rovelli's view:

Example 2 Enter third observer, ROV

We describe the situation from the point of view of a third observer, ROV. He has made measurements on S, O, and P in the past. On this basis he can say that the initial state of measured system S is a superposition $\sum \beta_i |A, i\rangle$ of eigenstates of an observable A corresponding to distinct eigenvalues, and that A will be measured twice, by two observers (measuring systems) O and P.[12] The pointer observable on both O and P is B, with eigenstates $\{|B, i\rangle\}$.

[12]It is part of ROV's knowledge, based on past measurements, that I and O each evolve freely after their measurement interaction, that there is no interaction between O and P, and that both A and the 'pointer-reading observable' B for O commute with the free Hamiltonians for S and O respectively.

Then, just calculating the time evolution on that basis, the combined system $S+O$ will be in dynamic state $\sum \beta_i (|A,i\rangle \otimes |B,i\rangle)$ relative to ROV at the end of the first measurement. At the end of the second measurement the dynamic state of $S+O+P$ relative to ROV will be $\sum \beta_i (|A,i\rangle \otimes |B,i\rangle \otimes |B,i\rangle)$.

By reduction, parts of the total system also have states relative to ROV at those times. As before, write $P[|A,i\rangle]$ for the projection on the ray containing $|A,i\rangle$, etc. At the end of the first measurement, the individual systems S and O are in states $[\sum |\beta_i|^2 P[|A,i\rangle]$ and $\sum |\beta_i|^2 P[|B,i\rangle]$ relative to ROV, respectively. At the end of the second measurement, the final state of $O+P$ relative to ROV, also calculated by reduction, is $\sum |\beta_i|^2 P[|B,i\rangle \otimes |B,i\rangle]$.

There is *no need* to carry out a *third* measurement, of the 'agreement observable' M, because it is predictable with certainty by ROV that he will get 1 if he does.

But suppose now that ROV asks himself what O and P found, and whether they found the same thing. Then he is asking a question that has no answer, for he cannot answer questions about their past given that he made no measurements on the basis of which he could answer those questions!

Now, of course, ROV can decide to make two new separate measurements on O and P, to see what they are registering now. So suppose he measures $I \otimes B$ on $O+P$ and gets value k. At this point he can make a prediction with certainty of what he will find if he then measures $B \otimes I$ on this system: for now the state of $O+P$ relative to ROV is the result of conditionalizing the one he had, on this result. He predicts with certainty that he will see the same pointer reading $|B,k\rangle$ on O.

Was k the value that O and P saw at that earlier time? At this point we have no basis for thinking that this question can make sense on Rovelli's view. There are no states of O, P relative to ROV which could be consulted to answer it.

So, to summarize: with a von Neumann mindset we insist that there must be a fact of the matter about what O and P saw, *tout court*, and that a fact of the matter is always enshrined in a definite quantum state. But in Rovelli's world that is not the way things are.

5 Can We Go Beyond the Resolution of These Puzzles?

What we have seen is that the puzzles one might have at first sight of Rovelli's account can be resolved. But the resolution leaves one still uneasy, for it hinges on the point that an observer O can register a measurement outcome—e.g. the answer 1 to question ?A—but this fact is not equivalent to O being in a particular physical state, whether relative to itself or relative to any other observer.

In other words there are elements of Rovelli's 'meta' description which may in particular cases not correspond to any information had by any observer, and hence *apparently* not describable in the language of quantum mechanics. One might be tempted to introduce the fiction that there is a 'universal observer' who knows what information is had (what answers have been registered) by each 'ordinary' observer. But this fiction can certainly not be admitted without ruining the story.

At the same time, in our reflections on what the observers register as measurement outcomes, we are targeting the very basis of Rovelli's understanding of quantum mechanics, and the very basis of the description of Rovelli's world:

> Quantum mechanics is a theory about the physical description of physical systems relative to other systems, and this is a complete description of the world. (Sect. II-C)

Drawing on Rovelli's favorite illustration of different frames of reference in Einstein's world, we are clearly tempted to ask: but *what relations are there between the descriptions that different observers give when they observe the same system?* Of course there can be no clue at all to an answer if we assume that there are no interactions at all between these distinct observers. But perhaps we can get a clue if we think of those distinct observers as themselves subject to observation by a third observer! Doing so need not be illegitimate if we recall that Rovelli is describing the general form that any ascription of states or observable-values can take, and that this is the form of information that an observer *could* have.

5.1 Rovelli's Symbolism for the Information Held, Simplified

Let us take a look back at how, in his 'meta' description, Rovelli introduces a symbolism to express the fact that a given system 'has' information about another one:

> If there is a maximal amount of information that can be extracted from the system, we may assume that one can select in $W(S)$ an ensemble of N questions Q_i, which we denote as $c = \{Q_i, i = 1, N\}$, that are independent from each other. There is nothing canonical in this choice, so there may be many distinct families $c, b, d, \ldots$ of N independent questions in $W(S)$. If a system O asks the N questions in the family c to a system S, then the answers obtained can be represented as a string that we denote as
>
> $$s_c = [e_1, \ldots, e_N]_c \tag{4}$$
>
> The string s_c represents the information that O has about S, as a result of the interaction that allowed it to ask the questions in c. (Section III-C)

The idea of a state of S relative to O enters now, because on the basis of this information, O can locate S in a finite subspace of the pertinent Hilbert space—even assign it a particular pure state represented by a vector in that space if the question-answer sequence was a maximally compatible one. This is what we describe informally in:

> (Form 1) O registers answer 1 to complete question $?A$, so S has state $|A, 1\rangle$ relative to O.

We observe now that there is in effect a *time order*: the order in which the questions are asked. (Only order in time will be regarded for now, not time metric.) The N questions in numbered line (4) appear in the order $1, \ldots, N$ so we can think of them as time-points, and can suggestively take them to indicate times $t_1, \ldots, t_N$. But then the less formal description of (Form 1) should be expanded to the form:

(Form 2) O registers answer e1 to complete question $?A(1)$ at time t_1, so S has state $|A(1), e1\rangle$ relative to O at $t_1, \ldots, O$ registers answer eN to complete question $?A(N)$, so S has state $|A(N), eN\rangle$ relative to O at time t_N.

Moreover, in general O will calculate evolved states for periods between measurements (compare the formulations by Groenewold where this is made explicit). So I suggest that we can therefore speak of an evolving relative state, as follows:

(Form 3) S has state $|\psi(t)\rangle$ relative to O, during the interval (t_1, t_N)

or, when we note only certain special moments in that interval, the following is an acceptable form of description:

(Form 3-FIN) S has states $|\psi(1)\rangle, \ldots, |\psi(N)\rangle$ relative to O, at times $t_1, \ldots, t_N$.

In some contexts it will be convenient to suppress the time reference, and just use Form 1, but in other contexts we will have to use the full form 3 or 3-FIN.

5.2 Concrete Example Retold as by ROV

As concrete example let us take the situation in Example 2, introduced in Puzzle 3, in which observers O and P were themselves subject to observation by an outside observer ROV, who started with the same initial information about $S + O$ that P had, but also information about P, so that he can foresee the sequence of two measurements that were displayed in the example.

I will now designate the initial time as t_0 and the ending times of the two measurement interactions as t_1 and t_2.

States Relative to ROV

a) The measured system S starts off in a state $\theta = \sum \beta_i |A, i\rangle$ relative to ROV, which is a superposition of eigenstates of an observable A corresponding to distinct eigenvalues.
b) A will be measured twice, by two measuring systems O and P. Each of O and P will be in the 'ready to measure' state relative to ROV to start, $|B, r\rangle$. The indicator states are $|B, i\rangle$ for eigenvalues i of A (which do not include r).

We assume that I and O each evolve freely after their measurement interaction, that there is no interaction between O and P, and that both A and the 'pointer-reading observable' B for O commute with the free Hamiltonians for S and O respectively.

c) The combined system $S + O$ will be in state $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle)$ relative to ROV at t_1, the end of the first measurement.
d) Similarly at that time, taking into account the as yet unchanging P, the state of $S + O + P$ relative to ROV will be the superposition $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle \otimes |B, r\rangle)$.
e) At t_2, the end of the second measurement the state of $S + O + P$ relative to ROV will be $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle \otimes |B, i\rangle)$.

Further States Relative to ROV

When we look at d) and e) above, we see that the state of S relative to ROV does not change after t_1, because the coefficients in the superposition do not change, even though the components do.

To show this, note that by reduction, we have states also for parts of the total system, namely S, O, P, relative to ROV. As before we write $P[|A, i\rangle]$ for the projection on the ray containing $|A, i\rangle$, etc. We deduce

f) ROV assigns to S all by itself an evolving *mixture* $\rho(S, ROV)(t)$ of the states $|\psi(t, i)\rangle$ such that:

$$\text{for } t < t_1 \text{ the state} \quad |\psi(t, i)\rangle = \theta = \sum \beta_i |A, i\rangle,$$
$$\text{for } t_1 \leq t \text{ the state} \quad |\psi(t, i)\rangle = |A, i\rangle$$

This mixture has as components the projections on these evolving vectors, one for each value i such that the coefficient β_i is not zero, and the weights are the 'squares' of those coefficients.

Note well, that *there is no change in this relative state at the second measurement time*, since in the superposition for the entire system, the values of B in O and P are the same in each component (that is, for every eigenvalue i such that β_i is not zero) from that moment on. Hence the weights in the mixture do not change from t_1 on.

5.3 ROV Observes Five Measurements

In fact, by the definition of von Neumann type measurements—entirely in terms of the quantum mechanical states and evolution operators (Hamiltonians)—there are *five* such measurements in the situation we described just now! The initially given measurements are:

a measurement of A by O ending at intermediate time t_1
a measurement of A by P ending at final time t_2

Both of these have pointer observable B, and the criterion they meet, to count as von Neumann measurements, is that the interaction is such that

(**vN Criterion**) beginning state $|A, k\rangle \otimes |B, r\rangle$ of system $S + O$ evolves into $|A, k\rangle \otimes |B, k\rangle$, where k is any eigenvalue of A; and a fortiori, beginning state $(\sum \beta_i |A, i\rangle) \otimes |B, r\rangle$ evolves into $(\sum \beta_i |A, i\rangle \otimes |B, i\rangle)$

and similarly for P. By the same token, there is also:

a measurement of A by O, also ending at t_2

where for simplicity we take A not to be time-dependent (if it is, the same holds, but the state of S relative to O evolves, in a way that O can calculate, and so adjust with time passing—no need, for our argument, to cover the general case). So O just keeps showing a value for A, and assigning the corresponding eigenstate to S, for all times from t_1 on.

But there is more. From the above it follows that the interaction between S and the total system $O + P$ is also the correlate of a measurement—in fact of three distinguishable measurements. For example, if we take $B \otimes I$ and $I \otimes B$ respectively as pointer observables on $O + P$, then the vN criterion is satisfied for times t_1 and t_2 respectively. So we have:

two measurements of A by $O + P$, ending at t_1 and t_2 respectively

To see this, in the story as told in terms of states relative to ROV, let us look at the overall evolution of the system, relative to ROV.

At the final time t_2, the complete system $S + O + P$ is in pure state $\sum \beta_i (|A, i\rangle \otimes |B, i\rangle \otimes |B, i\rangle)$ relative to ROV. By reduction the other states relative to ROV are:

$$S \text{ is in } \sum |\beta_i|^2 P[|A, i\rangle]$$

$$O \text{ and } P \text{ are both in } \sum |\beta_i|^2 P[|B, i\rangle]$$

$$S + O \text{ and } S + P \text{ are both in } \sum |\beta_i|^2 P[|A, i\rangle \otimes |B, i\rangle]$$

$$O + P \text{ is in } \sum |\beta_i|^2 P[|B, i\rangle \otimes |B, i\rangle]$$

Inspection shows that the vN Criterion is satisfied for the interactions I mentioned. But we can add one more: taking $B \otimes$ B as pointer observable, we also see $O + P$ engaged in a measurement that ends at the later time t_2. That is the fifth measurement which appears in this story of the states of these various systems relative to ROV, and their various evolutions.[13]

The reason it is important to note this is of course that observers gain information about systems only by measurement, and it is only if they gain information about systems that those systems have states relative to them. So now we can continue, in accordance with the meta-description of Rovelli's world, to see what states S has relative to O, P, and $O + P$.

States Relative to O, P, $O + P$

We can find the states of S relative to O, to P, and to $O + P$ for that interval, except that there will be some unknowns in it, namely the eigenstates that these observers assign to S on the basis of the measurements they make on it. (ROV makes no measurements on S, after the interval begins, that is why there are no similar unknowns in our calculation of $\rho(S, ROV)$.) So we arrive at:

g) Observer O assigns to S an evolving pure state $\rho(S, O)(t)$:

$$\text{for } t < t_1 \text{ the state } \rho(S, O)(t) = \theta = \sum \beta_i |A, i\rangle,$$

$$\text{for } t_1 \leq t \text{ the state } \rho(S, O)(t) = |A, m\rangle$$

[13] There can be no objection, it seems to me, to allow for trivial limiting cases: if O has absolutely no interactions with S through which information is gained, it is only a matter of bookkeeping if we say that then the state of S relative to O is the represented by the Identity operator—the 'informationless' statistical operator. This convention may at times smoothen the presentation, even if it is not really needed.

and here the value m is an unknown, it is the result that O registers as outcome of the measurement.

h) For P it is only a little more complicated: P assigns to S a mixture $\rho(S, P)(t)$ of the evolving pure states $\lambda(t, i)$ with weights $|\beta_i|^2$:

$$\text{for } t < t_1 \text{ the state } \lambda(t, i) = \theta = \sum \beta_i |A, i\rangle,$$
$$\text{for } t_1 \leq t < t_2 \text{ the state } \lambda(t, i) = |A, i\rangle,$$
$$\text{for } t_2 \leq t \text{ the state } \lambda(t, i) = |A, k\rangle$$

and here the value k, the outcome of P's measurement is unknown.

In the case of $O + P$ we see that it is an observer who makes two measurements, one precisely at the time of O's measurement, and one at the time of P's measurement, and finds respectively at that time the values r and s—two unknowns for us, as for ROV, for we have no basis or law on which to connect the outcomes of measurements by different observers, no matter how intimately they may be related. But just as did O, this observer does not assign a mixture, it assigns the pure state

i)

$$\begin{aligned}\rho(S, O + P)(t) &= \text{the state } \theta = \sum \beta_i |A, i\rangle, \quad \text{for } t < t_1 \\ &= \text{the state } |A, r\rangle, \quad \text{for } t_1 \leq t < t_2 \\ &= \text{the state } |A, s\rangle, \quad \text{for } t_2 \leq t\end{aligned}$$

with r and s as the unknowns.

We would like to see what constraints could be added that would ensure concordance between the states of a system S relative to different observers such as O, P, and ROV—and here it will be pertinent for us that we have to keep also $O + P$ in view.

So now, finally, I'm going to propose an addition to Rovelli's account.

5.4 Additional Postulate Relating Relative States

Additional Postulate. For any systems S, O, P, witnessed by ROV:

- the state of S relative to O (if any) cannot at any time be orthogonal to the state of S relative to $O + P$ (if any), and
- the state of S relative to P (if any) similarly cannot be orthogonal to the state of S relative to $O + P$ (if any),
- and the state of S relative to any of these cannot be orthogonal to the state of S relative to ROV,
- (and so forth for larger composite situations).

Here too the words "if any" are needed for generality; but in our example, the three systems do assign states to S. We may note again that the case of pure states is very special, and in general (as opposed in our examples here) the relational states will be mixed—and there is no associated 'ignorance interpretation' of mixtures. The requirement of non-orthogonality is rather restrictive for pure states, but of course always less so for mixtures.

What could be the motivation and intuitive warrant for this postulate, within the point of view of relational quantum mechanics? As Rovelli presented his own motivation he refers to the example of Einstein's methodology in the creation of relativity theory in just the same way that the Copenhagen physicists took their inspiration from that episode. The inspiration took the form of a certain kind of moderate empiricism: nothing was to be attributed to how nature itself is or proceeds beyond what is manifested in measurement outcomes. Thus the overriding case for the denial that certain observables really do have simultaneous sharp values when not measured is precisely that there is no measurement procedure to reveal that possibility. More precisely, no configuration of values of observables is to be postulated for unmeasured nature unless there is a state in which measurement would show that configuration as outcome, with certainty.

In the quantum case, where transition probabilities are zero precisely when the relevant states are orthogonal, we can encapsulate this idea in the

Slogan: Born probability $= 0 \rightarrow$ NO!

So consider how the situation looks to ROV. When ROV contemplates measurements on these systems, to see if the pointer observables of O, P, $O+P$ could be in disagreement with each other at the pertinent times, the calculation of the Born conditional probability for this will be zero. So, to follow the above suggestion as to how to conceive of the un-measured world, ROV will conceive of the relations between what the subsystems register accordingly. The idea that any assertion about what happens in nature must have cash value in what we can expect to detect, measure, or observe is strong in the Copenhagen tradition, even if contradicted by hidden variable enthusiasts. It seems to me that it echoes precisely the sort of inspiration that both the Copenhagen theorists and Rovelli derive from Einstein's reasoning when he introduced relativity.

So how is this inspiration honored by our Additional Postulate? If we now look back to our description of the evolving states of S, through the relevant time interval, relative to these three observers, we see the following pure state assignments:

$\rho(S, O)(t)$ remains the same from t_1 on, namely $|A, m\rangle$

$\rho(S, P)(t)$ is a mixture until t_2 when it becomes $|A, k\rangle$

$\rho(S, O+P)(t)$ is $|A, r\rangle$ for times from t_1 on, till it becomes $|A, s\rangle$ at t_2

For different values of m, k, r, s, those vectors are mutually orthogonal, since they are all eigenvectors of the same operator. So the second and third line immediately tell us that $k = s$. But the first and third line tell us that $m = r$ when we attend to t_1, and similarly that $m = s$, when we attend to t_2. So all these numbers are after all the same.

Result: the evolving states of S relative to the observers O and P are not the same to begin, but they are the same once P makes its A-measurement on S, sometime after O did (with no disturbance of A intervening meanwhile).

Supposing ROV to be knowledgeable of Relational Quantum Mechanics *thus extended*, what can he know even though he has made no measurements during or after that interval, on O and P?

He knows that what they found as outcomes of their measurements were indeed the same.

He already knew on the basis of Quantum Mechanics alone that *if he made a measurement to check on such agreement* he would get the answer YES with certainty. But now, calculating from the same previous measurement results that constitute his initial information, but using also Additional Postulate, he deduces that the agreement he would find with certainty if he measured was indeed already there.

This pleasing result, I have to emphasize, is found only by adding this additional postulate concerning how *the information registered* by components of a composite system engaged in several measurements are related to each other. So I cannot pretend that this harmony between the information obtained by different interacting observers follows from what Rovelli presented.

But Rovelli did not go into the question of whether there are three-way connections between information that can be had by observers in such a larger situation. This additional postulate was phrased so as to add only to the general form in which information can be had by different systems in a complex situation—without ever assigning any quantum mechanical states that are observer-independent.

I submit that the addition is consistent with Rovelli's account, and does not go essentially beyond what Rovelli allows himself in the 'meta' description in which he couches his depiction of the world of quantum mechanics. For it remains that all that has been provided—once we recognize the holism in composite situations involving many interactions—is an answer to "what is the general form of a description of the world from the vantage points of different observers?"

6 Relational EPR

Laudisa [9] and Smerlak and Rovelli [12] have examined how the Einstein–Podolski–Rosen situation can be regarded or modeled within Relational Quantum Mechanics. They do not entirely agree in their approach. Here I shall show how the situation fares if my Additional Postulate is accepted. The result appears to be different from what is favored by Rovelli, though it does not seem to affect the empirical content of the resulting formulation of quantum mechanics.

Let S be a two-part system $\alpha + \beta$ (such as a photon pair in singlet state), in a superposition of correlated states $\uparrow \otimes \downarrow$ and $\downarrow \otimes \uparrow$. The arrows are eigenvalues of observable A.

Observers $P1$ and $P2$ respectively measure $A \otimes I$ and $I \otimes A$ with pointer observable B. ROV has information on initial states and dynamic process

$$P1 \text{ gets } \uparrow \text{ or } \downarrow \ldots \text{ the state of } \alpha \text{ relative to } P1 \text{ is } |\uparrow\rangle \text{ or } |\downarrow\rangle$$

$$P2 \text{ gets } \uparrow \text{ or } \downarrow \ldots \text{ the state of } \alpha \text{ relative to } P1 \text{ is } |\uparrow\rangle \text{ or } |\downarrow\rangle$$

$$P1 + P2 \text{ gets } \uparrow\uparrow \text{ or } \uparrow\downarrow \text{ or } \downarrow\downarrow \text{ or } \downarrow\downarrow \ldots \text{ the state of } \alpha + \beta \text{ relative to}$$
$$P1 + P2 \text{ is } |\downarrow\rangle \otimes |\uparrow\rangle \text{ or } (|\uparrow\rangle \otimes |\uparrow\rangle \text{ or } (|\downarrow\rangle \otimes |\downarrow\rangle \text{ or } (|\uparrow\rangle \otimes |\downarrow\rangle$$

For $P1 + \alpha + P2 + \beta$ ROV assigns at the measurement time a superposition of

$$(|B, 1\rangle \otimes |\uparrow\rangle) \otimes |B, 2\rangle \otimes |\downarrow\rangle$$

and

$$(|B, 2\rangle \otimes x|\downarrow\rangle) \otimes (|B, 1\rangle \otimes |\uparrow\rangle)$$

This implies that ROV assigns to $\alpha + \beta$ a mixture of $(|\uparrow\rangle \otimes |\downarrow\rangle)$ and $(|\downarrow\rangle \otimes |\uparrow\rangle)$.

By the Additional Postulate it follows that the state of $\alpha + \beta$ relative to $P1 + P2$ must be one of these, thus ruling out two of the possibilities noted above. And then $P1 + P2$ will assign to α and β separately either $|\uparrow\rangle$ and $|\downarrow\rangle$ respectively or $|\downarrow\rangle$ and $|\uparrow\rangle$ respectively. But then, again by the Postulate, the states of α and β relative to $P1$ and to $P2$ respectively cannot be the same, on pain of orthogonality to what they are relative to $P1 + P2$.

Have we arrived at 'spooky' non-locality? We need to be worried by possible conflict with the sentiment so clearly expressed in [12]:

> There is no operational definition of observer-independent comparison ... of different observers' information ...: the information of different observers can be compared only by a physical exchange of information between the observers.

Can ROV, in our story (including the Additional Postulate), compare the two states of $\alpha + \beta$ relative to $P1$ and $P2$ before measuring $P1$ and $P2$ at the end?

YES and NO!

ROV can know that $P1$ and $P2$ did not register the same value for A. But to know anything about *which* values they did register, ROV would have to make measurements. So ROV can *predict* no more than someone who has not heard the additional Postulate, but only that *if he measured both* he would find different registered values, which is predictable with no reliance on the Additional Postulate.

We can define a function of the outcomes registered by $P1$ and $P2$, which takes value 1 if the outcomes are the same and value 0 if they are different. It seems then that ROV can know the value of this defined quantity, without having measured it. But the defined quantity has value 1 if and only if the states of α and β relative to $P1$ and $P2$ respectively are either $|\uparrow\rangle$ and $|\downarrow\rangle$, or $|\downarrow\rangle$ and $|\uparrow\rangle$. That this is so I do not think follows in the original Relational Quantum Mechanics. Therefore this could be counted as running contrary to the above cited sentiment.

But I would like to suggest that it may count as a reason for the suggested Additional Postulate. For otherwise we leave open the possibility that the state of $\alpha + \beta$ relative to $P1 + P2$ is $(|\uparrow\rangle \otimes |\downarrow\rangle$, for example, although the states of α and β relative to $P1$ and $P2$ respectively are both $|\uparrow\rangle$!

Even if we were to insist that " S has state ... relative to O" can only have a truth value related to a further observer ROV (and not be true or false 'absolutely') this same difficulty would appear when ROV is in the picture.[14] But there is much to

[14] The suggestion here would be, it seems to me, a radicalization of the original Relational Quantum Mechanics, but perhaps closer to the initial intuitions than what I have worked with here. However,

explore here yet, including the most radical view, namely that even what the states relative to any observer are must itself be relative to an observer.

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Appendix: An Auxiliary Formalism

Finally, ***solely as an aid to the imagination***, we can add some auxiliary symbolism, as follows. We note that O registers an answer in an entirely physical way, in that this measurement involves—and requires—a correlation of the measured observable A on S with a 'pointer' observable B on O. Hence, if we wish to mark that correlation, we have a final variant on Form 1:

$$\text{(Form 1-bis) } O \text{ has } [[|B, 1\rangle]]$$

This looks deceptively like a state attribution to O, which it is not—it is meant as an equivalent to Form 1, when it is known that O's pointer observable is B, and no more. We can think of this as an encoding of the information O has about S in the spirit of Rovelli's remark:

> Let me then take a lexical move. I will from now on express the fact that q has a certain value with respect to O by saying: O has the "information" that $q = 1$. (Section II-E)

In this passage, q is an observable that pertains to a specific system, the one that is measured by O, so despite the surface form this is still a relational statement. A specific example will have a projection on a subspace for q, and a still more specific example will have this projection operator one-dimensional, in which case to say '$q = 1$' is the same as ascribing a specific pure state (to the measured system S, relative to O).

In the October 2006 symposium at the University of Provence, Aix, Carlo Rovelli voiced some suspicion of this auxiliary symbolism, so I undertook to restate the argument without reliance on that device.

Using the auxiliary symbolism, however, we have an equivalent alternative to the Additional Postulate:

> If a composite observer $X + Y$ has $[[\boldsymbol{\phi}]]$, while X has $[[\boldsymbol{\xi}]]$ and Y has $[[\boldsymbol{\xi}']]$ then $\boldsymbol{\xi}$ is possible relative to reduced state $\#\,\boldsymbol{\phi}$ and $\boldsymbol{\xi}'$ is possible relative to reduced state $\boldsymbol{\phi}\#$.

Written in this way, one can see a formal relationship—though well short of agreement throughout—with the modal interpretation (CVMI) as defended in [14]. There,

though worth exploring further, I see it as difficult to sustain, given its obvious danger of either regress or circularity—well, perhaps worth exploring precisely because of such danger!

and in the there cited section of my 1991, it is possible to see just how the last line in the Additional Postulate ("and so forth for larger composite situations") would need to be elaborated in detail.

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